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Name of college - S.S. college, J-Bad

Deptt - Mathematics
Topic - polar Equation of conic Section
Class - B.Sc I (Hon+Sub)
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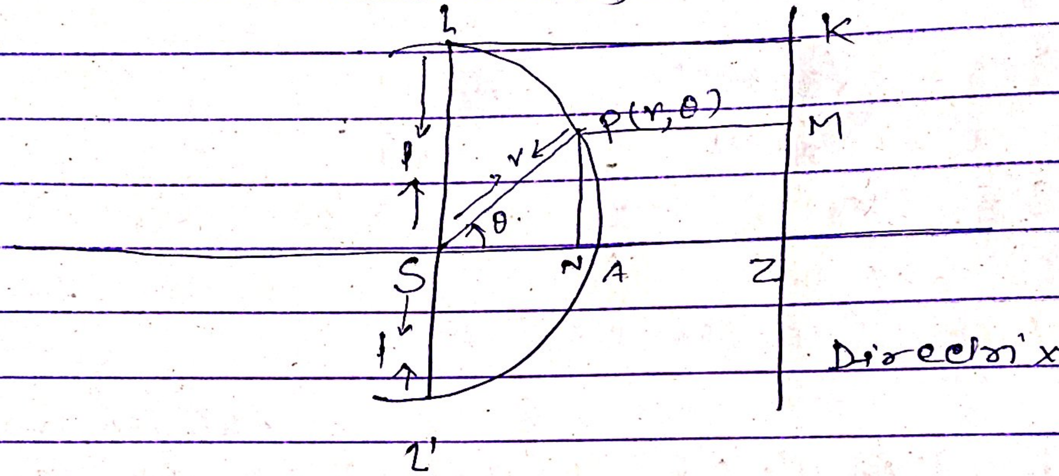
Polar Equation of Conic Section



Let S be the focus ZM the directrix and e = eccentricity of the conic.
Draw SZ perpendicular to the directrix. Let here SZ be considered as the initial line and S the focus as pole.
Let LSL' = 2l = latus rectum
∴ SL = SL' = l

Let P(r, θ) be any point on the conic.
then SP = r ∠PSA = θ
By Definition of the conic,
$\frac{SL}{LK} = e$

$\Rightarrow SL = e\,LK$

$= e\,SZ$

Again $\frac{SP}{PM} = e$

$SP = e\,PM$

$= e\,NZ$

$= e[SZ - SN]$

$\Rightarrow r = eSZ - eSN$

$= l - e\frac{SN}{SP} \times SP$

$= l - er.\frac{SN}{SP}$ From right-angled ΔSNP $\frac{SN}{SP} = \cos\theta$

$= l - er\cos\theta$

$\Rightarrow er\cos\theta + r = l$

$\Rightarrow r[1 + e\cos\theta] = l$

$\Rightarrow \frac{l}{r} = 1 + e\cos\theta$

this is the required equation of the conic

Remarks $\rightarrow$ If ZS produced is taken as the initial line then θ will be $-\theta$

Therefore equation of conic in this case is

$l = r[1 + e\cos(-\theta)]$

$= r[1 - e\cos\theta]$

~~In case~~ the case of parabola, $e = 1$

$\therefore \frac{L}{r} = r[1 - \cos\theta]$

2. Equation of the chord through two given points on the conic

$$\frac{l}{r} = 1 + e\cos\theta.$$

Since the Equation of the Conic is

$$\frac{l}{r} = 1 + e\cos\theta \quad \text{--- (1)}$$

Let P and Q be the two points on the conic whose vectorial angles are α and β respectively

then the polar Co-ordinate of P $\left(\frac{l}{1+e\cos\alpha}, \alpha\right)$

and the polar Co-ordinate of Q $\left(\frac{l}{1+e\cos\beta}, \beta\right)$

The Equation

$$\frac{l}{r} = A\cos\theta + B\sin\theta. \quad \text{--- (2)}$$

represent a st. line passing through two points. Where A, B are two constants

If (2) passes through P and Q then

$$1 + e\cos\alpha = A\cos\alpha + B\sin\alpha \quad \text{--- (3)}$$

$$\Rightarrow (A - e)\cos\alpha + B\sin\alpha - 1 = 0 \quad \text{--- (3)}$$

Similarly.

$(A - e)\cos\beta + B\sin\beta - 1 = 0$ ——— (10)

From (3) and (4)

using cross multiplication

$$\frac{A-e}{-\sin\alpha+\sin\beta} = \frac{B}{-\cos\beta+\cos\alpha} = \frac{1}{\cos\alpha\sin\beta-\sin\alpha\cos\beta}$$

$$\therefore A - e = \frac{\sin\beta-\sin\alpha}{\cos\alpha\sin\beta-\sin\alpha\cos\beta}$$

$$\Rightarrow A = \frac{\sin\beta-\sin\alpha}{\cos\alpha\sin\beta-\sin\alpha\cos\beta} + e$$

$$= \frac{2\cos\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}}{\sin(\beta-\alpha)}$$

$$= \frac{\cancel{2}\cos\frac{\alpha+\beta}{2}\cancel{\sin\frac{\beta-\alpha}{2}}}{\cancel{2}\cancel{\sin\frac{\beta-\alpha}{2}}\cos\frac{\beta-\alpha}{2}} + e$$

$$\cos\frac{\beta+\alpha}{2}\sec\frac{\beta-\alpha}{2} + e$$

and $B = \dfrac{\cos\alpha-\cos\beta}{\sin(\beta-\alpha)} = \sin\dfrac{\alpha+\beta}{2}\sec\dfrac{\beta-\alpha}{2}$

Putting two obtained

values of A and B in (2)
we get

$$\frac{l}{r} = \left[\cos\frac{\beta+\alpha}{2}\sec\frac{\beta-\alpha}{2} + 0\right]\cos\theta$$
$$+ \sin\frac{\beta+\alpha}{2}\sec\frac{\beta-\alpha}{2}\sin\theta$$

$$\Rightarrow \frac{l}{r} = \sec\frac{\beta-\alpha}{2}\cos\left[\theta - \frac{\beta+\alpha}{2}\right] + e\cos\theta$$

This is the required equation of chord joining

P(α) and Q(β)

If vectorial angles of p and q be respectively α−β and α+β,

Then the equation is

$$l/r = \sec\beta\cos(\theta-\alpha) + e\cos\theta$$

This is the required Equation of the chord PQ.